7.4.1 Differentiation

Differential calculus is based on the notion of rate of change. The notion appears in such functions as growth rate, relative growth, rate of reaction and others. The operation of finding the derivative of a function is called differentiation.

so if $y = x^n$ then $\frac{dy}{dx} = nx^{n-1}$ (7.12)

The differential coefficient of a constant is zero. For instance, if y = a, where a is any constant then d/dx of a = 0.

other special cases

if $y = \frac{1}{x}; \frac{dy}{dx} = -x^{-2}$

$y = \frac{1}{x^2}; \frac{dy}{dx} = (-2)x^{-3}$

$y = \sqrt{x}; \quad \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

Box 7.4

Problem : differentiate the function $y = x^2$

Solution : Here x is the independent variable and y the dependent variable and we find the derivative of y with respect to x.

then since $y = x^2$

$y + \delta y = (x + \delta x)^2$

or $y + \delta y - y = (x + \delta x)^2 - x^2$

or $\delta y = x^2 + 2x\delta x - (\delta x)^2$

or $\frac{\delta y}{\delta x} = 2x + \delta x$

when limit $\left(\frac{\delta y}{\delta x}\right)$ = limit $(2x + \delta x)$ or $\frac{\delta y}{\delta x} = 2x$

$\delta x \longrightarrow 0$ $\quad$ $\delta x \longrightarrow 0$

so if $y = x^n$, $\frac{\delta y}{\delta x} = nx^{n-1}$ (7.12)

The derivative of a composite function is equal to the derivative of the outer function times the derivative of the inner function. For example if y = f(u) and u = F(x) then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad (7.13)$$

Variables can be interchanged so that the dependent variable can be converted to the independent variable and vice-versa:

$$\frac{dx}{dy} = \frac{1}{dy/dx} \quad (7.14)$$

For instance if y = 3x – 4, then dy/dx = 3. The inverse function is $x = \frac{1}{3}(y+4)$.

Hence, dx/dy = 1/3.

7.4.2 Higher Order Derivatives

If y is a function of x, its differential coefficient dy/dx is, in general, another function of x, which is capable of differentiation again. The differential coefficient of dy/dx is called the second differential coefficient of the original function and is denoted by $\frac{d}{dx}\left(\frac{dy}{dx}\right)$ or $\frac{d^2y}{dx^2}$.

The third differential coefficient of y with respect to x is denoted by $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$ or $\frac{d^3y}{dx^3}$ and so on (box 7.5)

Box 7.5

Problem : If $y = 3x^4 - 2x^3 + 5x^2 - 6x + 1$, find $\frac{d^4y}{dx^4}$

Solution : $\frac{dy}{dx} = 12x^3 - 6x^2 + 10x - 6$

$\frac{d^2y}{dx^2} = 36x^2 - 12x + 10$

$\frac{d^3y}{dx^3} = 72x - 12$ and $\frac{d^4y}{dx^4} = 72$